

Answer the following questions. Each question counts 5 points.

Calculators are not allowed

1) Evaluate each of the following limits, if it exists:-

$$i) \lim_{x \rightarrow 1} \frac{\sqrt{x+1} - \sqrt{2x}}{x^2 - x}$$

$$ii) \lim_{x \rightarrow 0} \left( \frac{\tan^3 x}{(x^4 + 5x^3)} \right)$$

2) Use  $\epsilon$  and  $\delta$  definition of the limit to show that :

$$\lim_{x \rightarrow 1} (-3x + 5) = 2$$

3) Let  $f$  be a function defined on  $(-\infty, +\infty)$ . If  $16 \leq f(x) \leq x^4$ , for all  $x$ ,

$$\text{find } \lim_{x \rightarrow 8} f(x)$$

4) Find the vertical and horizontal asymptotes (if any) for the graph of

$$f(x) = \frac{x}{2\sqrt{4x^2 - 1}}$$

5) State the Intermediate Value Theorem. Then show that the equation

$$x^5 + 3x^3 + 2x - 8 = 0, \text{ has at least one real root.}$$

6) Find the constants  $A$  and  $B$  so that the function

$$f(x) = \begin{cases} Ax - 1, & \text{if } x < 1 \\ 2x^2 + B, & \text{if } x \geq 1 \end{cases}$$

will be differentiable at  $x = 1$

7) Find  $y'$ , if  $y = \frac{\sqrt{x} \sin x}{\csc x}$

8) Use the definition of the derivative to find  $f'(x)$  for  $f(x) = x^3$ .

Good Luck